Appendix - Let S_H and I_H be the number of susceptible and infectious individuals in a human population. Let also S_m and I_m be number of susceptible and infectious individuals, respectively, in a mosquito population. We can write (Massad et al. 2001):

(1)

$$\frac{dI_H}{dt} = aN_m \frac{I_m}{N_m} b \frac{S_H}{N_H} - \gamma I_H$$

$$\frac{dI_m}{dt} = \exp(-\mu\lambda) aS_m(t-\tau) \frac{I_H}{N_H} - \mu I_m$$

In the first equation of system (1) a is the average number of bites a mosquito inflicts in the human population, so that aN_m is the total number of bites per unit of time and b is the fraction of those bites that are actually infective for humans. Humans recover from infection with rate γ. Finally, only a fraction S_H/N_H of the infective bites is in susceptible individuals, resulting in new infections.

In the second equation of system (1) the extrinsic incubation period is represented by τ. Therefore, $aS_m(t-\tau)$ is the number of bites per unit time given by susceptible mosquitoes in the instant $(t-\tau)$. A fraction I_H/N_H of those bites is in infected humans, producing infected mosquitoes, a fraction exp(-μτ) of which survives the extrinsic incubation period τ to become infective for the rest of their lives. Mosquitoes are assumed to die by natural causes at a rate μ.

Dividing the first equation of system (1) by N_H (total human population) and the second by N_m (total mosquito population) we have:

(2)

$$\frac{di_H}{dt} = mai_m bS_H - \gamma i_H$$

$$\frac{di_m}{dt} = \exp(-\mu\lambda) aS_m(t-\tau) i_H - \mu i_m$$

where $m = N_m/N_H$ and the lower case letters represent proportions.

At the beginning of an outbreak, we can assume that $S_H \cong 1$ and $S_m(t-\tau) \cong 1$ to get the linearized system

(3)

$$\frac{di_H}{dt} = mai_m b - \gamma i_H$$

$$\frac{di_m}{dt} = \exp(-\mu\lambda) ai_H - \mu i_m$$

whose general solution is

(4)

$$i_H = c_H \, exp(\lambda t)$$

$$i_m = c_m \, exp(\lambda t)$$

Taking the derivative of equations (4) and substituting the result in equations (3) we get:

(5)

$$\lambda = \frac{1}{2} [-(\mu + \lambda) \pm \sqrt{((\mu + \gamma)^2 - 4\mu\gamma + 4ma^2 b \exp(-\mu\tau))}]$$

Remembering that R_0 is

(6)

$$R_0 = \frac{ma^2 b \exp(\mu \lambda)}{\gamma \mu}$$

Therefore, from equations (5) and (6) we obtain (Massad et al. 2001):

(7)

$$R_0 = 1 + \frac{\lambda^2 + \lambda(\mu + \gamma)}{\gamma \mu}$$